Precalc Warm Up # 2-2

Find without using a calculator. Make a conjecture about how $\log_a b$ and $\log_b a$ are related.

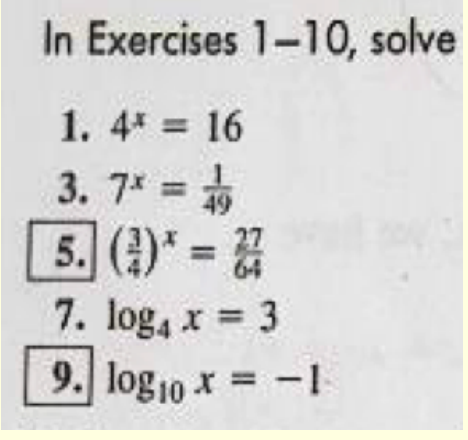
1. $\log_3 9$ 2. $\log_9 3$ 3. $\log_9\left(\frac{1}{3}\right)$ 4. $\log_{1/3} 9$

Solve for x

5. $\log_5(x+2)=-1$ 6. $\log_x e=-2$ 7. $\ln x = e$

8. $e^{x+4}=-5x-2$ 9. $3^x = 17$ 10. $(2x+7)^{x^2-4}=1$

HW Questions: p. 284



In Exercises 1–10, solve

1. $4^x = 16$
3. $7^x = \frac{1}{49}$
5. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$
7. $\log_4 x = 3$
9. $\log_{10} x = -1$

In Exercises 11–16, apply the inverse properties of ln x and e^x to simplify the given expression.

11. $\ln e^{x^2}$

13. $e^{\ln(5x+2)}$

15. $e^{\ln x^2}$

Solve:

20. $8(10^{3x}) = 12$

$x \approx 0.051$

26. $1000e^{-4x} = 75$

$x \approx 0.648$

In Exercises 49–64, solve

49. $\ln x = 5$

51. $2 \ln x = 7$

53. $2 \ln 4x = 0$

55. $\log_{10}(z - 3) = 2$

57. $\ln x + \ln(x - 2) = 1$

$\ln(x^2 - 2x) = 1$

$e = x^2 - 2x - e$

$-e$

$x = \frac{2 \pm \sqrt{4 - 4(-e)}}{2}$

$x = 1 \pm \frac{\sqrt{4(1+e)}}{2}$

$1 \pm \sqrt{1+e}$

$1 + \sqrt{1+e}$

59. $\log_{10}(x + 4) - \log_{10} x = \log_{10}(x + 2)$

61. $\ln x + \ln(x + 3) = 1$

63. $\ln x^2 = (\ln x)^2$

let $y = \ln x \rightarrow$

$2 \ln x = (\ln x)^2$

$2y = y^2$

$0 = y^2 - 2y$

$0 = y(y - 2)$

$\ln x = 0 \quad \ln x - 2 = 0$

$e^0 = x \quad \ln x = 2$

$x = 1 \quad x = e^2$

In Exercises 65 and 66, find the time required for a $1000 investment to double at interest rate r, compounded continuously. Solve for t in the exponential equation $2000 = 1000e^{rt}$.

$2000 = 1000e^{0.085t}$

$2 = e^{0.085t}$

$\ln 2 = 0.085t$

65. $r = 0.085$

70. In a group project in learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be

$$P = \frac{0.83}{1 + e^{-0.2n}}.$$

After how many trials will 60% of the responses be correct?

$$\frac{.60}{1} = \frac{0.83}{(1 + e^{-0.2n})}$$

$$\frac{0.6}{0.6}(1 + e^{-0.2n}) = \frac{0.83}{0.6}$$

$$1 + e^{-0.2n} = \frac{83}{60} - \frac{60}{60}$$

$$-1$$

$$e^{-0.2n} = \frac{23}{60}$$

$$\frac{\ln\left(\frac{23}{60}\right)}{-0.2} = \frac{-0.2n}{-0.2}$$

$n \approx 4.8$

≈ 5 trials

Tools

A logarithm is an exponent.

Definition: $y = \log_a x$ iff $x = a^y$

a must be positive and $\neq 1$, and X must be positive

Basic Log Properties:

$\log_a a = 1$ $\quad \log_a 1 = 0$ $\quad \log_a a^x = x$

$\log_a cd = \log_a c + \log_a d$ $\quad a^{\log_a x} = x$

$\log_a (c/d) = \log_a c - \log_a d$

$\log_a c^m = m(\log_a c)$

Review:

Find all solutions:

$$(x^2 + 2x - 7)^{(x+2)} = x^2 + 2x - 7$$

Review:

Find all solutions:

$$(x^2 + 2x - 7)^{(x+2)} = x^2 + 2x - 7$$

(Any #)^1 = itself

$x + 2 = 1$

$x = -1$

$(1)^{\text{Any \#}} = 1$

$x^2 + 2x - 7 = 1$

$x = -4, 2$

$(-1)^{\text{odd}} = -1$

$x^2 + 2x - 7 = -1$

$x = -1 \pm \sqrt{7}$

check $x + 2$ is odd

$-1 \pm \sqrt{7} + 2$ not odd

Any # > 0

$\bigcirc = \bigcirc$

$x^2 + 2x - 7 = 0$

$x = -1 \pm 2\sqrt{2}$ → only $x = -1 + 2\sqrt{2}$

check $x + 2 > 0$

$-1 \pm 2\sqrt{2} + 2 \overset{?}{>} 0$

$1 \pm 2\sqrt{2} \overset{?}{>} 0$

Find all solutions: $(x^2 - 10)^{(x + 6)} = 1$

Find all solutions: $(x^2 - 10)^{(x + 6)} = 1$

$(\text{Any } \# \neq 0)^0 = 1$

$x + 6 = 0$

$\boxed{x = -6}$

check $x^2 - 10 \neq 0$

$36 - 10 \neq 0$ ✓

$1^{\text{Any } \#} = 1$

$x^2 - 10 = 1$

$x^2 = 11$

$\boxed{x = \pm\sqrt{11}}$

$(-1)^{\text{even}} = 1$

$x^2 - 10 = -1$

$x^2 = 9$

$x = \pm 3$

check $x + 6$ even

$\pm 3 + 6$ not even

11

Simplify

1. $\log_4 2 + \log_4 32$

2. $\log 200 - \log 2$

3. Given that $\log_a x = 1.3652$, find

   a. $\log_a x^2$

   b. $\log_a x a^3$

Simplify

1. $\log_4 2 + \log_4 32$

$\log_4 (2 \cdot 32)$

$\log_4 64$

3

2. $\log 200 - \log 2$

$\log \left(\frac{200}{2}\right)$

$\log_{10} 100$

2

3. Given that $\log_a x = 1.3652$, find

   a. $\log_a x^2$

$2(\log_a x)$

$2(1.3652)$

2.7304

   b. $\log_a x a^3$

$\log_a x + \log_a a^3$

$1.3652 + 3$

4.3652

Solve. Check for extraneous solutions!

4. $\log_4(x-5) - \log_4 x = 2\log_4 7$

5. $\log_4(x-5) - \log_4 x = 7$

Solve. Check for extraneous solutions!

4. $\log_4(x-5) - \log_4 x = 2\log_4 7$

$\log_4\left(\frac{x-5}{x}\right) = \log_4 49$

$\frac{x-5}{x} = 49$

$x \neq -\frac{5}{48}$ // extraneous

5. $\log_4(x-5) - \log_4 x = 7$

$\log_4\left(\frac{x-5}{x}\right) = 7$

$4^7 = \frac{x-5}{x}$

$x \neq \frac{-5}{16,383}$ // extraneous

HW: SL book

p. 227 #1-6 LC, 8

Quiz Thurs
PC: 4.1- 4.4
SL: 7.1

*You may use one of my graphers.